B.Tech Degree I & II Semester Examination in Marine Engineering June 2011

MRE 101 ENGINEERING MATHEMATICS I

Time: 3 Hours Maximum Marks: 100

(All questions carry ***EQUAL*** marks)

I. (a) Evaluate $\underset{x\to 0}{lt}\ \frac{xe^x - \log(1+x)}{x^2}$.

(b) State Rolle's theorem. Verify the theorem for $\frac{\sin x}{e^x}$ in $(0, \pi)$.

(c) If $y = e^{a\sin^{-1}x}$, prove that $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - \left(n^2+a^2\right)y_n = 0$.

OR

II. (a) Prove that the radius of curvature at any point of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is three times the length of perpendicular from the origin to the tangent at that point.

(b) Find the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$.

III. (a) If $u = \left(x^2+y^2+z^2\right)^{-1/2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

(b) If $u = \sin\left(x/y\right)$, $x = e^t$ and $y = t^2$. Find $\frac{du}{dt}$ as a function of t.

OR

IV. (a) Verify Euler's theorem for the function $f(x,y) = \frac{x^2\left(x^2-y^2\right)^3}{\left(x^2+y^2\right)^3}$.

(b) If $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J' = \frac{\partial(x,y)}{\partial(u,v)}$, then prove that $JJ' = 1$.

(c) Discuss the maxima and minima of $f(x,y) = x^3y^2(1-x-y)$.

V. (a) Derive the standard equation of a hyperbola.

(b) Find the condition for the straight line $\ell x + my + n = 0$ to be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(c) Show that the locus of point of intersection of perpendicular tangents to the parabola $y^2 = 4ax$ is the directrix $x + a = 0$.

OR

VI. (a) Find the vertex, focus directrix and length of latus rectum of the ellipse $x^2 + y^2 - 8x - 6y - 31 = 0$.

(b) Find the equation of hyperbola passing through (2, 3) and has straight line $4x + 3y - 7 = 0$ and $x - 2y - 1 = 0$ as asymptotes.

VII. (a) Derive the formula for $\int_0^{\pi/2} \sin^n x$.

(b) Find the area of the segment cut off from the parabola $x^2 = 8y$ by the line $x - 2y + 8 = 0$.

(c) Find the entire length of cardioid $r = a(1+\cos\theta)$.

OR

VIII. (a) Change the order of integration and hence evaluate it

$$\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dxdy$$

(b) Evaluate $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x+y+z)\,dxdydz$

IX. (a) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$.

(b) A line makes angles $\alpha, \beta, \gamma, \delta$ with diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

(c) If $\bar{a}x\left(\bar{b}\times\bar{c}\right) = \left(\bar{a}\times\bar{b}\right)\times\bar{c}$, show that $(\bar{a}\times\bar{c})\times\bar{b} = 0$.

OR

X. (a) Find the directional directive of $xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of vector $i + 2j + 2k$.

(b) Show that the vector $\left(x^2+xy\right)i + \left(y^2+xy\right)j$ is irrotational.

(c) Prove that $\left[\bar{a}\times\bar{b},\ \bar{c}\times\bar{d}, \bar{e}\times\bar{f}\right] = \left[\bar{a}\ \bar{b}\ \bar{d}\right]\left[\bar{c}\ \bar{e}\ \bar{d}\right] - \left[\bar{a}\ \bar{b}\ \bar{c}\right]\left[\bar{d}\ \bar{e}\ \bar{f}\right]$.

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